Beyond the Binary: Dexterous Teaching and Knowing in Mathematics Education

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This paper identifies binary oppositions in the discourse of mathematics education and introduces a binary-epistemic model for (re)conceptualising these oppositions and the epistemic-pedagogic problems they represent. The model is attentive to the contextual relationships between pedagogically relevant binaries (e.g., traditional/progressive, student-centred/teacher-centred, discovery/transmission, constructivist/behaviourist) and epistemically relevant binaries (e.g., concrete/abstract, pure/applied, interpretivist/positivist, subjective/objective) that operate in mathematics classrooms. The premise of this paper is that ways of knowing mathematics (i.e., epistemologies) are actualised in ways of teaching mathematics (i.e., pedagogies), and vice-versa. The binary-epistemic model describes oppositional, equipositional and parapositional ways of knowing and teaching in relation to these binaries. We argue for a more a relational-contextual or *parapositional* approach to binary polarities that have otherwise proven divisive in mathematical discourse. In the context of the new Australian Curriculum, we illustrate epistemically differentiated ways of teaching measurement in a Year 5 mathematics classroom.

Keywords: epistemological development • pedagogy • mathematics • teacher education • curriculum

Introduction

> One plausible hypothesis is that formal education plays an important role in the development of students' beliefs about the nature of mathematical knowledge and learning. (Muis, 2004, p. 339)

The popular national discourse on literacy and numeracy in teacher education programs reveals deep divisions and tensions over the nature of mathematical knowledge and relatedly, effective pedagogies for mathematical education. For example, the Australian Curriculum makes the point that:

> Mathematics provides students with essential mathematical skills and knowledge in Number and Algebra, Measurement and Geometry, and Statistics and Probability … It encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences. (ACARA, 2013b)

Arguably, recognition of opposition and engagement with division can lead to cooperation and deeper understanding, or at least, better management of conflict and difference. Illustratively, Thomas (2011) laments that "divisions in the [Australian mathematics] community that arose in the 1990s have taken a terrible toll", yet notes of the US *math* wars, "while it may have been destructive at the time, it has led to much greater cooperation between the various bodies" (p. 137). This paper provides a conceptual framework for understanding and engaging these divisions and tensions in the context of the Australian Mathematics Curriculum.

The recent rollout of the Australian Curriculum provides a timely opportunity for reflection on the pedagogical approaches that interpret and inform the curriculum. As noted in the National Curriculum Board's (2009) document, *Shape of the Australian Curriculum: Mathematics*, the "content and organisation of a national mathematics curriculum is based on some pedagogical assumptions" (p. 14). The coordination of these different and seemingly contradictory assumptions presents a "wicked problem" for mathematics educators. A "wicked problem" (Rittel & Webber, 1973) has no definitive formulation, no immediate or ultimate test of solution, no clear contextual delineation, and is open only to (*re*)solving and (*re*)solution rather than final objective solutions. The concept is sometimes used interchangeably with "ill-structured problems" (Mitroff et al., 2004), "messes" (Ackoff, 1993), and "social messes" (Horn, 2001). Mathematics teachers in fluid and dynamic contexts make complex pedagogical decisions to maximise students' learning in highly differentiated classrooms.

Much of the "wickedness" of divisiveness lies with the difficulty of accurately representing and relating these pedagogical divisions, let alone acknowledging and articulating their epistemic dimensions. Our premise is that teachers' epistemic approaches to wicked problems in the domain of mathematics will affect their pedagogical stances and ultimately, their students' dispositions towards mathematics. Student experience of teaching and learning can affect their *disposition* towards mathematics and by extension, the role and status of mathematics in society at large. As De Corte, Op 'T Eynde, Depaepe and Verschaffel (2010) note in *Personal Epistemology in the Classroom*, "Fostering in students the acquisition of positive or availing mathematics-related beliefs is at present regarded as an important goal of mathematics education" (p. 294). Accordingly, we argue that there is an important relationship between ways of knowing (i.e., epistemologies) and ways of teaching (i.e., pedagogies) in the context of mathematics education (e.g., Beswick, 2005; Cobb, Wood, & Yackel, 1990; Opdenakker & van Damme, 2006; Wilson & Cooney, 2002). The seemingly abstract tensions between epistemological polarities (e.g., *concrete/abstract*, *interpretivist/positivist*, *analytic/synthetic*, *subjective/objective*, *a priori/a posteriori*) find concrete expression in the mathematics classroom and affect students' learning.

The coordination of such common polarities presents a wicked problem in many domains of knowledge, not the least of which include mathematics and pedagogy. This paper offers a binary-epistemic model based on a general consensus of theory and research in personal epistemology to help represent and inform mathematics teachers' epistemic-pedagogic choices. For example, is there a semantic affinity between, (a) teacher-centred approaches, direct instruction, explicit instruction, guided instruction, transmissive teaching, and skills-based pedagogies within a behaviourist framework, and (b) positivistic, objectivistic, abstract, reductive and analytic epistemologies? Similarly, is there a pedagogical affinity between (a) student-centred approaches, inquiry-based learning, discovery-based learning, scenario-based learning, and collaborative learning within a constructivist framework, and (b) interpretivist, subjectivist, concrete, holistic, and synthetic epistemologies? Relatedly, are these respective epistemologies (e.g., positivist/interpretivist) and pedagogies (transmission/discovery) *necessarily* oppositional?

Ultimately, we propose a more relational and contextual or *parapositional* approach to these binaries that is consistent with reflective (King & Kitchener, 2002), relational and contextual (Reich, 2002), evaluative (Kuhn & Weinstock, 2002; Tabak & Weinstock, 2008), and paradoxical ways of knowing (i.e., epistemologies). However, first it is important to illustrate the relationship between pedagogy and epistemology and their related binaries in relevant literature. The following section provides a background to some of the epistemic tensions that influence pedagogical choices in the context of mathematics education.

Epistemic-pedagogic Constructs in Mathematics Education

Epistemic-pedagogical affinities and divisions are borne out in popular and scholarly commentary. The currency of pedagogical divisions related to the Australian Curriculum is illustrated in a Queensland context by criticism of the traditional pedagogical nature of *Curriculum to Classroom* (C2C). For example, in the context of the mathematics C2C, Kennedy, O'Neill and Devenish (2011) argue:

> However the approach to content knowledge in C2C differs from ACARA's approach in a subtle, but important way. Factual knowledge, routine questions and efficiency dominate, frequently at the expense of flexibility. In a textual analysis of 57 lessons across grades 1–7 we found that 86% of the actions associated with teachers fit within a traditional teaching approach where teachers demonstrated, modelled or explained particular content to students, followed by time for practice questions that were very similar in nature to that which the teacher had just shown. (p. 10)

Accordingly, curriculum cannot escape pedagogical expression and such expression helps to identify related epistemic divisions and affiliations. For example, Slavich and Zimbardo (2012) identify a pedagogical affinity between active learning, student-centered learning, collaborative learning, experiential learning and problem-based learning, and argue:

> At the deepest conceptual level is the fact that these approaches share similar theoretical roots. At the heart of all types of active and student centered learning, for example, is the constructivist notion that students generate knowledge and meaning best when they have experiences that lead them to realize how new information conflicts with their prevailing understanding of a concept or idea. (p. 6)

They contrast these collective approaches with "a lecture-based approach, in which instructors assume the role of 'sage on the stage' and dictate information to students, who have little role in shaping the experience" (p. 3). Remillard (2005) identifies the epistemic-pedagogic link even more explicitly in his characterisation of positivist and interpretivist approaches to mathematics curricula. He notes "incongruence between the epistemological assumptions underlying program goals and how teachers tended to carry them out" (p. 218).

Interestingly, Askew et al's (2010) meta-analysis of over 500 studies concluded that mathematical proficiency is "much more closely linked to cultural values than to specific mathematics teaching practices" (pp. 33–34). However, it could be argued that there is a relationship between a culture's values, its epistemic climate, and its dominant mathematics teaching practices. For example, Wong (2007) notes that the teacher-centred approach to mathematics is a culturally dominant model in Hong Kong, grounded in the belief that it can help students to move between the concrete and the abstract. Needless to say, as in any other domain of life and study, in the meeting of cultures across borders and within mathematics' staffrooms and classrooms, there are epistemic conflicts.

There are many examples of common binary-epistemic structures present in mathematics discourse. For example, in relation to a *cognitive/affective* binary, Seah and Wong (2012) claim that "the various cognitive approaches to improving mathematics teaching and learning alone have not been sufficient in effecting real, sustained improvements in how mathematics is taught and learnt in the school classroom" (p. 35). Izmirli (2011) identifies "seemingly dichotomous categories" (p. 28) including *teaching/learning*, *theory/application*, *logic/intuition*, *practical/theoretical* and *concrete/abstract*. In the context of Australian literacy and numeracy for Indigenous students, Noel Pearson (2011) provides an example of an epistemic-pedagogic approach that argues for a deliberate developmental school-based imbalance in the context of a broader balance between the *skills and knowledge/creativity and critique* binary:

> Creativity will not be killed by giving priority to basic skills … All diversions and impediments—every excuse, every suggestion that harm will come from such a focus, every appeal to 'balanced' approaches to basic skills mastery—have to be removed. (Pearson, 2011, p. 108)

Pearson is fiercely combative when it comes to the pedagogies of the progressive left and argues for a need to "sail on an uneven keel" to *right* the historical wrongs of the left. For Pearson, the left is characterised by its focus on students' self-esteem, feelings, and a resistance to testing: "Seen in terms of the Old Left, this kind of critical pedagogy is just the teaching of false consciousness, powered by moral vanity" (p. 110). The right is characterised by "basic skills mastery … practice makes perfect … and a solid grounding in knowledge" (pp. 108–109). Person's "uneven keel" analogy hints at the epistemic dimension of a wicked pedagogical problem that warrants a more analytical exploration of the dynamics behind such metaphors.

In relation to an *interpretivist/positivist* binary, Remillard (2005) identifies one of the more common epistemic transpositions in mathematics (*fixed/dynamic*): "knowledge that is indeed dynamic and 'unsteady,' like the messy work of proving a theorem, is often portrayed as static and tidy when presented as a proof in a book" (pp. 230–231). Such transpositions can cause epistemic conflict between teachers, or between teachers and policy makers for two reasons, either (a) because they have interpretivist or positivist dispositions, or (b) because of contextual constraints that make these dispositions difficult to overcome (e.g., time for a teacher or space for an author to interpret and contextualise a mathematical proof). Similarly, Otte (1986) notes the difficulty of treating mathematical texts *subjectively or objectively* as "a permanent problem not to be solved once and for all" (p. 175). Arguably, Otte's observation extends to mathematical knowledge and pedagogy more broadly. The subjective/objective, interpretivist/positivist distinctions present a wicked problem to be solved and (*re*)solved in mathematics classrooms.

Another wicked epistemic-pedagogic problem in the mathematics classroom is related to the potential for competition between mathematical epistemologies and pedagogical epistemologies. For example, Nardi, Biza and Zachariades (2012) observe that:

> … teachers' acceptance, scepticism or rejection of students' mathematical utterances—as expressed in their evaluation of these utterances and their feedback to the students—does not have exclusively mathematical (epistemological) grounding. Their grounding is broader and includes a variety of other influences, most notably of a pedagogical, curricular, professional and personal nature. (p. 162)

Thus, dispositional disciplinary epistemologies must also be reconciled with personal pedagogies and epistemologies. This is true for teachers and students. For example, De Corte, Op 'T Eynde, Depaepe and Verschaffel (2010) suggest that students' beliefs about the learning and teaching are domain-specific and contextual, with the domain of mathematics dominated by "students' epistemological beliefs about the nature of mathematical knowledge as certain and fixed" (pp. 296–297).

This *fixed/dynamic* knowledge binary has affiliations with another common binary in mathematics: *real/applied*. Several studies reveal that most students perceive mathematics as unrelated to the real world (Schoenfeld, 1992) and about facts, figures, and procedures (Muis, 2004), with mathematics teachers dominantly represented by students as authoritarian and lacking common sense (Picker & Berry, 2000). While the causes of these perceptions are complex, we suggest that they are at least related to the epistemic identities of mathematics that are implicit in its pedagogical expressions. Furthermore, our contention is that the wicked problem of reconciling epistemic-pedagogic "opposites" in mathematics is best (*re*)solved with a relational and contextual approach to knowledge.

Implicitly Relational and Contextual Approaches

The relational and contextual approach to epistemic-pedagogic tensions in mathematics is implicit in several existing treatments. Perhaps most eloquently, Izmirli (2011) writes: "

> The proposed epistemological revision would ameliorate our synthesis of the seemingly dichotomous categories such as teaching and learning, theory and application, logic and intuition by demonstrating the unity of practical (concrete) mathematical knowledge and its theoretical (abstract) counterpart. (p. 28)

Similarly, in the context of mathematics education, Seah and Wong (2012) argue that "there is no one-size-fits-all pedagogical approach to effective teaching and learning in schools . . . [and] teachers will need to be prepared to negotiate conflicting cultural and pedagogical values in their attempts to deliver effective lessons" (pp. 33–34). Walshaw and Anthony (2008) utilise an "evolving systems network" and suggest, "In this system, the teacher and the students are mutually constituted through the course of interactions" (p. 521).

Slavich and Zimbardo (2012) forward a "supraordinate framework called transformational teaching" (p. 2) and suggest that various constructivist methods "are synergistically related and, when used together, maximize students' potential for intellectual and personal growth" (p. 1). In a British context, Askew et al's (1997) study of 90 primary school teachers and over 2000 students found that a challenging curriculum, high expectations of low-achieving students, and a focus on students' mathematical learning were more important than overall teaching style (i.e., transmission or discovery-based). This study raises an important dynamic in the epistemic conceptualisation of different pedagogies—the battle for semantic affiliation. For example, advocates of transmission and discovery orientated pedagogies often characterise themselves positively and the other negatively in terms of such characteristics (i.e., transmission = high expectations; discovery = engaged students). A relational-contextual approach attempts to (de)contextualise and then (re)contextualise such binary oppositions to show that in context, students can also be engaged through transmissive teaching and challenged with high expectations through discovery learning.

Summarily, mathematics teaching is often conceptualised through binary constructs such as fixed/dynamic, objective/subjective, discovery/transmission, student-centred/teacher-centred, constructivist/behaviourist, and interpretivist/positivist; with collective affinities between left binary constituents and right constituents, and common binary oppositions between the collective left and right. Our two assumptions are (a) that there are deep epistemic-pedagogic affinities revealed in everyday discussion and debate about the teaching of mathematics, and (b) the epistemic dimension of these affinities and oppositions needs more explicit consideration towards a more robust and (re)solvable dialogue about the teaching of mathematics.

Our thesis is that a framework that is evaluative, relational-contextual, and grounded in theory and research on epistemological development, can move the pedagogical dialogue beyond polemic and opposition, without diluting real and important differences.

Binary-epistemic Development to Inform Pedagogical Praxis

Epistemological development relates to changes in ways of knowing and beliefs about the nature of knowledge across the lifespan. There are many models of epistemological development that have grown out of research in educational psychology (Table 1). These models are complemented by more literary-symbolic descriptions of epistemological development present in most, if not all domains of knowledge (e.g., literary studies and philosophy). As is widely acknowledged in literature on epistemological development, "The trajectories of these models suggest a general

transition from a dualistic perspective of knowledge to a more relativistic stance and ultimately to a contextual, constructivist perspective on knowing" (Wildenger, Hofer & Burr, 2010, p. 222).

Table 1
Stage Similarities in Models of Epistemological Development

Theory	Perry (1970)	Belenky et al. (1986)	Baxter & Magolda (1992)	King & Kitchener (1994)	Kuhn & Weinstock (2002)	West (1996)
Stage	Dualism	Received	Absolute	Pre-reflective	Absolutism	Absolute
	Multiplicity	Subjective knowing	Transitional	Quasi-reflective	Multiplism	Personal
	Relativism	Procedural	Independent			Rules-based
	Commitment in relativism	Constructed knowledge	Contextual	Reflective	Evaluativism	Evaluative

To recall, our assumption is that teacher's ways of knowing (i.e., epistemologies) affect their ways of teaching (i.e., pedagogies). Many researchers (e.g., Arredondo & Rucinski, 1996; Brownlee and Berthelsen, 2008; Chan & Elliot, 2004) suggest that teachers' epistemological beliefs influence their approaches to teaching. For example, Brownlee and Berthelsen (2008) claim that teachers with objectivist beliefs favour more transmissive teacher-centred approaches, and teachers with evaluativistic beliefs are more likely to adopt constructivist and student-centred approaches to their teaching. We would argue that this characterisation is too semantically simplistic to be helpful in the dialogue between teachers and teacher-educators who subscribe to different pedagogies, or who draw from a range of pedagogies in context. Arguably, an evaluativistic approach is not one that favours student-centred over teacher-centred pedagogies a priori, but one that (a) evaluates the strengths and weaknesses of each approach *in context*; (b) acknowledges the relationality and interdependence of the two approaches; and (c) acknowledges the degrees of difference and semantic shades of grey between the approaches. While agreeing with Brownlee and Berthelsen's (2008) argument that transmission pedagogies tend to reflect early epistemologies, our approach is that there are contexts and instances in which effective transmission can enable discovery; discovery can be communicated through effective transmission; and discovery can be realised in the presence of transmission. In other words, the appearance of epistemic tendencies early in development does not negate their more sophisticated *re*alisation and *re*creation in later development. This is a well-recognised dynamic in epistemological development related to the subjective/objective binary. Acknowledging Kuhn and Weinstock's (2002) work on the subjective/objective distinction, Leah et al (2010) write:

> The absolutist sees knowledge from an objective perspective, the multiplist takes a subjective view, and finally, the evaluativist achieves a mature balance of the two, coordinating a personal and subject frame of knowing with an awareness of how knowledge can be verified. (p. 222-223)

Based on this general movement, we propose a model of binary-epistemic development as a framework for teachers to consider and evaluate epistemic-pedagogic relationships in their own contexts.

A Model of Binary-epistemic Development

The binary-epistemic model (Figure 1) represents development in ways of knowing in relation to binary constructs. The model proposes three general positions (*Oppositional*, *Equipositional* and

Parapositional) of teacher development in relation to common epistemic-pedagogic binaries. It uses a seesaw metaphor to represent archetypal positions and developments in relation to binary constructs (i.e., A/B). The paraposition is most encompassing of other positions while being attentive to contextual factors that require evaluation and adaptive selection between positions. This is represented by contextually shifting polarities (black and white) and syntheses (grey) in relation to a moving fulcrum.

Oppositional ways of knowing and teaching. Oppositional ways of knowing and teaching (A > B, A < B) represent dispositions to choose a particular position in opposition to its antithesis, regardless of context. Thus, Oppositional Position A is a dichotomising position that reflects a relative tendency to approach wicked problems from the epistemic-pedagogic perspective of A (i.e., knowledge as concrete, interpreted, subjective, local, synthetic, holistic, dynamic; teaching as student-centred, inquiry-based, collaborative, and constructivist), with an opposition to B. Whereas, Oppositional Position B represents a dichotomising position that reflects a relative tendency to approach wicked problems from the epistemic-pedagogic perspective of B (i.e., knowledge as abstract, positive, objective, universal, analytic, reductive, fixed; teaching as teacher-centred, transmissive, reproductive, individualistic and behaviourist), with an opposition to A.

Equipositional ways of knowing and teaching. Equipositional ways of knowing and teaching (A = B; A + B = C) are characterised by a tendency to approach all problems with an equal measure of A and B, regardless of context. One form (Equipositional A = B) reflects an early dialogical tendency to approach wicked problems using relatively polarised epistemic perspectives of A and B in equal measure, regardless of context. Another form (Equipositional C) reflects an early dialectical tendency to approach wicked problems using an equalising "middle position" representing a balanced combination of epistemic-pedagogic perspectives A and B, regardless of context.

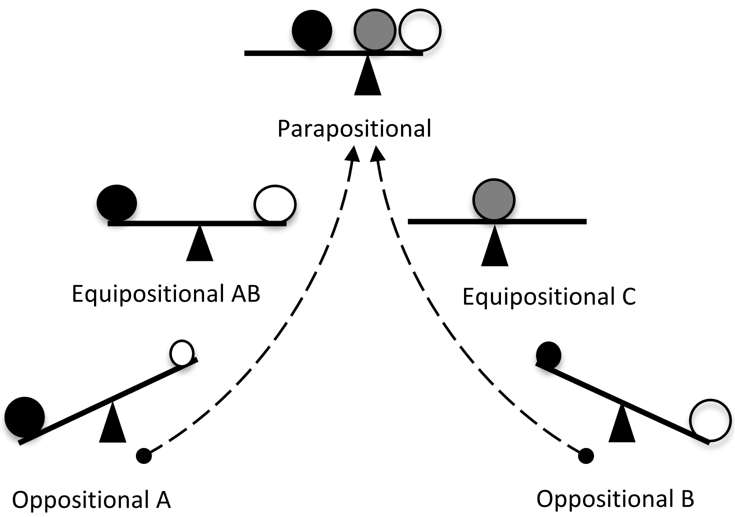

Figure 1. Binary-epistemic development towards *parapositional* ways of knowing and teaching.

Parapositional ways of knowing and teaching. Parapositional ways of knowing and teaching (A <=> B) are characterised by (a) an understanding of the interdependent and relational nature of different pedagogies, and (b) the relational, contextual, and evaluative application of these pedagogies for effective learning. The position reflects a relative tendency to approach wicked pedagogical problems using relational perspectives of A and B in a manner that is dependent on context. For example, contextual variables could include students' age, cultural background and disposition towards the subject, curriculum directives, school imperatives, and teacher's pedagogical disposition, as well as pragmatic concerns (e.g., time, space and material resources). The position represents the ability to draw on previous positions (e.g., Oppositional A or B) with an understanding of the fluidity of context, and with an adaptive ability to change positions or transform contexts, accordingly to maximize learning. The prefix *para* is chosen for its multiple meanings including beside (e.g., *para*llel), beyond (e.g., *para*normal), union (e.g., *para*biosis), and opposition (e.g., *para*chute). However, these understandings are always intentionally, rather than arbitrarily selected and applied. Parapositional teachers appreciate that the salience and prevalence of some binaries may actually signal authentic *para*doxes that can help to expand rather than contract the range of pedagogical possibilities. Arguably, some epistemic-pedagogic debates are unnecessarily produced and sustained when a paradox is treated as a clear-cut opposition that can be resolved once and for all with a big enough or long enough empirical study. Rather, a teacher with a parapositional disposition uses empirical research to inform pedagogical decisions, but with an acute awareness of contextual limitations and the dangers of assessing the art of pedagogy with its scientific dimension, and vice-versa. Teachers with this epistemic-pedagogic dexterity are able to navigate freely but not aimlessly between the art and science of teaching; they are characterised by a seemingly effortless and unified flow within context and between contexts. The following section illustrates the differences between these positions in relation to the teaching of measurement in a Year 5 mathematics classroom.

Illustrations of Praxis

This section illustrates binary-epistemic positions in relation to pedagogical choices. The illustrations represent archetypal dispositions and positions in order to provide conceptual clarity for more nuanced location of pedagogical divisions and differences in specific classrooms. Like grid points on a map they help communicate epistemic-pedagogic positions and dispositions. We propose that this communication facilitates more effective navigation of the complex terrain of mathematics education.

The illustrations relate to a content descriptor from the Year 5 content strand *Measurement and Geometry* in the *Australian Curriculum: Mathematics*, "Calculate the perimeter and area of rectangles using familiar metric units" (ACARA, 2013, ACMMG109). This content descriptor is sequentially located between the Year 4 content descriptor, "Compare objects using familiar metric units of area and volume" (ACMMG290) and the Year 6 content descriptor, "Solve problems involving the comparison of lengths and areas using appropriate units" (ACMMG137). The following illustrations of epistemic-pedagogic choices are also provided and discussed with the curriculum rationale and structure in mind. For example, the Australian Curriculum rationale for mathematics states:

> The curriculum anticipates that schools will ensure all students benefit from access to the power of mathematical reasoning and learn to apply their mathematical understanding creatively and efficiently. The mathematics curriculum provides students with carefully paced, in-depth study of critical skills and concepts. It encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences. (ACARA, 2013, para. 4)

Furthermore, the proficiency strands that describe the development and exploration of curriculum content are summarised as:

> *Understanding:* Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas.
>
> *Fluency:* Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily.
>
> *Problem Solving:* Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively.
>
> *Reasoning:* Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. (ACARA, 2013)

Together, the rationale, content scope and sequence, and proficiency strands provide a rich background for exploring the epistemic-pedagogic dimension of mathematics teaching. The following scenarios involve five fictional teachers (HM, DS, RE, SG, AZ).

Oppositional A (A>B)

Introduction: This position is characterised by epistemologies that embrace subjective, relativistic, integrative, synthetic, a posteriori, experiential, interpretivist, phenomenological, holistic, and constructivist ways of knowing; and pedagogies that embrace student-centred, discovery, intrinsic, collaborative, inquiry-based, concrete and applied ways of teaching for learning.

A teacher with an epistemic-pedagogic disposition to these ways of knowing and teaching may approach the content descriptor (ACMMG109) as follows.

Description: HM is determined that "her children" will learn to love mathematics and see mathematical relationships and representations present in all of nature. Her class is often outside and she is rarely seen at the photocopier in the morning preparing worksheets. The class set of mathematics textbooks and the children's exercise books have had little use as HM wants the children to "learn from experience and not from books". HM tends to use the curriculum content descriptors and elaborations incidentally, such that they are engaged if, and when, they are relevant to the children's more authentic and immediate concerns as community and global citizens.

And so it is that when preparing an organic garden for a class-to-community project, HM considers the relevance of the content descriptor "Calculate the perimeter and area of rectangles using familiar metric units" (ACMMG109). Among other projects drawing on multiple learning areas, HM asks if any of the children would like to take responsibility for working out the best area needed to plant 10 tomato plants in the garden, and the length of mesh fencing they would need to surround the garden area. A group of five children volunteer and HM lets them have unstructured time to find out what they need to know. The children are asked to work out a budget for the fencing and tomato plants and are told that they can help purchase the mesh for homework and plant the tomato plants during lunchtime. HM allows the children to choose a way to describe how they solved the problem (e.g., write a narrative) and then recommend a grade for their work based on the process they used to solve the problem and the effectiveness of the final product. HM emphasises that they must work together as they will all receive the same grade for their project. In preparing for a Curriculum-in the-Classroom presentation to fellow staff, HM demonstrates how she has integrated different learning areas (e.g., Mathematics and

English) and content descriptors (e.g., ACMMG109) through the community garden project under the broader umbrella of the Sustainability cross-curriculum priority.

Oppositional B (A<B)

Introduction: This position is characterised by epistemologies that embrace objective, universal, analytical, a priori, abstract, reductive and rational ways of knowing; and pedagogies that embrace teacher-centred, transmissive, instructional, extrinsic, and rote ways of teaching for learning.

A teacher with an epistemic-pedagogic disposition to these ways of knowing and teaching may approach the content descriptor (ACMMG109) as follows.

Description: DS is determined that by the end of each unit "his students" will have the basic content knowledge outlined in the curriculum to solve a range of mathematical problems and move to the next level of conceptual sophistication required in Year 6. DS despairs at the "wishy-washy, lefty" pedagogies of some of his colleagues and prides himself on the quality and rigour of his teaching as justified by his students' excellent standardised test results year after year. His students do a scheduled session of English, Science, History and Mathematics each day, with little integration or deviation. DS's lessons are strictly curriculum-based and he systematically works his way through the scope and sequence of content descriptors and elaborations. He sees his job and the role of education to "teach the content" and demonstrate it, so that the children can apply it in their own time and in their own contexts and through their own interests outside of school hours. DS's students can often be heard chanting rules and formulas, or working individually and silently on textbook exercises. DS has a progress chart on the wall that represents students' achievements as measured by the weekly content test in each learning area. The children's books are filled with equations and worksheets, marked and scored in bold red pen.

And so it is that on Monday after lunch, DS's class files in to find a worksheet on the desk and a content descriptor and formulae written on the whiteboard:

> Calculate the perimeter and area of rectangles using familiar metric units.
> Rectangle: (Area) A = L x W. (Perimeter) P = 2L + 2W.

DS has the class read the descriptor and formulae in unison and then write them down in their exercise books. He then proceeds to explain the terms and demonstrate the formulae using a range of rectangles drawn on the whiteboard. The students then work individually and in silence to complete the worksheet DS has provided. The worksheet contains a series of rectangles of varying dimensions, and requires students to work out the area and perimeter. After a set amount of time, the children swap papers and grade each other's work as DS reads out the answers. When the papers are returned DS asks for a show of hands to indicate results and sets homework of re-completing all incorrect responses and an extra textbook exercise for those who got more than five wrong. The following day, homework is checked for completion and accuracy, students are individually asked to recite the formulae and calculate the response to impromptu questions given by DS on the area and perimeter of various abstract rectangles. By the end of the week DS is confident that he knows which children know and can apply the formulae.

Equipositional (A = B)

Introduction: This position is characterised by an abstractly equal balance of (a) epistemologies that embrace subjective, relativistic, integrative, synthetic, a posteriori, experiential, interpretivist, phenomenological, holistic, and constructivist ways of knowing; and pedagogies that embrace student-centred, discovery, intrinsic, collaborative, inquiry-based, concrete and

applied ways of teaching for learning; and (b) epistemologies that embrace objective, universal, analytical, a priori, abstract, reductive and rational ways of knowing; and pedagogies that embrace teacher-centred, transmissive, instructional, extrinsic, and rote ways of teaching for learning.

A teacher with an epistemic-pedagogic disposition to this way of knowing and teaching may approach the content descriptor (ACMMG109) as follows.

Description: RE's philosophy is characterised by a "balance in all things" approach. As much as is possible she equally, almost mechanically, divides her class time between different learning areas, and different pedagogical approaches and does not deviate from this division. For example, RE divides the school day into discrete blocks of learning for each core learning area. She also tries to divide session time equally into "teacher time" and "student time" and alternates which time is used in the first part of the session. Teacher time tends to consist of direct instruction through oral transmission and demonstration of concepts. For students, this time mostly involves listening, repeating and copying. Student time tends to consist of collaborative problem-based project work related to the concepts. For the students, this time mostly involves discussing, discovering and creating.

And so it is that on Tuesday morning, RE's Year 5 class begins a mathematics session by listening to an explanation of the content descriptor "Calculate the perimeter and area of rectangles using familiar metric units". They copy definitions of key terms (i.e., area, perimeter, rectangle), rote learn formulae and then individually complete three pre-set questions requiring students to measure the area and perimeter of different rectangles. On the half hour, RE signals "student time", whereupon the children work in groups with a computer to find a global application of the measurement of area and perimeter and a personally relevant application of the same.

Towards the end of the session the children are invited to share their ideas with the rest of the class. For example, Ari shares that his group found that farmers might measure the perimeter of paddocks in the shape of rectangles to find out how much fencing they need, and Ellen shares that her parents told her about area when choosing correct bedding sizes (e.g., single, double, queen) for her room.

Equipositional (A + B = C)

Introduction: This position is characterised by a "middle position" synthesis of approaches that reduces the use of either pedagogical or epistemic polarities. This equipositional form defines itself against polarities by emphasising the exclusive value of "the Golden Mean". Thus, epistemologies that embrace (a) subjective, relativistic, integrative, synthetic, a posteriori, experiential, interpretivist, phenomenological, holistic, and constructivist ways of knowing; and pedagogies that embrace student-centred, discovery, intrinsic, collaborative, inquiry-based, concrete and applied ways of teaching for learning; and (b) epistemologies that embrace objective, universal, analytical, a priori, abstract, reductive and rational ways of knowing; and pedagogies that embrace teacher-centred, transmissive, instructional, extrinsic, and rote ways of teaching for learning, are both shunned in equal measure for a middle position that synthesises the two.

A teacher with an epistemic-pedagogic disposition to this way of knowing and teaching may approach the content descriptor (ACMMG109) as follows.

Description: SG would refer to himself as a "teacher of the middle ground", whereas colleagues tend to characterise him as "fence-sitter". As much as is possible he attempts to find the common ground of different pedagogies and combine them into a single approach.

And so it is that on a Wednesday afternoon SG's Year 5 class enters the room to find pieces of grid paper with centimetre intervals on each desk. On another handout is the formula for finding the area of a rectangle, the formula for the perimeter of a rectangle, and an example of each. The students are then paired and taken into the school playground where they are given the task of recording the perimeter and area of any rectangles they can find. SG moves between the pairs during the session to explain the content knowledge that students are applying. Towards the end of the lesson the students share and compare their items and answers on the whiteboard, facilitated by SG. For example, Owen and Oonagh share their perimeters and areas in centimetres for a paving brick, an outdoor seat and a table surface. Kelly and Tariq share their perimeters and areas for the handball court, which took them the whole session to measure.

Parapositional (A<=>B)

Introduction: This position is characterised by a sort of pedagogical dexterity that can evaluate and select one or more ways of teaching and knowing from a range of possibilities, with a differentiated knowledge of context, individual student needs and practical limitations. The selection is not haphazard or driven by a need for equality without knowledge of context. It is appreciative of the abstract relational equality of polarities in teaching and knowing, but recognises that context can demand particular choices, that can change over time for the most effective student learning. Such teaching is conscious of the abstract paradoxes between subjective and objective, concrete and abstract, student-centred and teacher-centred; and yet it is informed rather than paralysed by them, in contexts that require real pedagogical choices and actions. The parapositional teacher is conversant with many binary positions (e.g., subjective/objective, interpretivist/ positivist, analytic/synthetic, teacher-centred/student-centred). However, understanding the relationality of the binary constituents, they can be reluctant to identify a priori with one or another approach, yet proficient in selecting, using and justifying a particular approach effectively in context.

A teacher with an epistemic-pedagogic disposition to this way of knowing and teaching may approach the content descriptor (ACMMG109) as follows.

Description: AZ has taught for many years. Throughout those years she has seen many programs and initiatives come and go. AZ recognises something cyclic, almost pendulum-like about the pedagogical and curricular swings and tug-of-wars that manifest in academia, education departments, school administrations and classrooms. However, rather than succumb to a debilitating cynicism, AZ balances a healthy scepticism of "quick-fix, one-size fits all" initiatives with a graduate's enthusiasm for trying new approaches and "seeing what works" with individual students. AZ has learned to apply the best of different pedagogical approaches, even polarised approaches, and avoid the worst, with a mindfulness of classroom context and a genuine interest in individual students. In mathematics, as in all learning areas, AZ wants to foster a mathematical disposition that is subjectively felt by the students and related to deep conceptual knowledge and an understanding of its applications. AZ demonstrates an epistemic-pedagogic fluidity and flow that breaks down static oppositions between concrete and abstract, subjective and objective, transmission and discovery, constructivism and behaviourism. Arguably, it is this pedagogical fluidity and dexterity that enables many of AZ's students to develop "the confidence to use the familiar to develop new ideas, and the "why" as well as the "how" of mathematics" (National Curriculum Board, 2009, p. 6).

And so it is that on Wednesday morning AZ's Year 5 class chatters expectantly, each holding a rectangular object brought from home the night before or chosen from around the classroom. AZ tries to involve students' home-lives but is mindful that some students may be marginalised

in the process. Marcus has a book, Marcia a shoebox, and Demi has a doormat. AZ centres herself in front of the students, waits for silence, greets the class and asks a few children to hold up their object and describe it. Then, AZ poses a question, "What do all of your objects have in common? What's the same about them?" The children offer some answers until one of them remarks, "they all have rectangles".

AZ then links the children's objects to many more things (concrete and abstract local and global) that have rectangles. By the end of the explanation the children have contributed almost twenty more rectangular objects. AZ uses the interactive whiteboard to display pictures of some of the objects the children mention and then gets individual children to outline the rectangle with a marker so that all children can identify the abstract shape. Finally, AZ calls out different groups of children to line up in order of the size of their object's rectangle. Having given students the opportunity to understand intuitively the relative size of different rectangles, AZ reveals some notes on the whiteboard and challenges the students to deepen and sharpen their ability to measure rectangles.

> Calculate the perimeter and area of rectangles using familiar metric units
> Rectangle: (Area) A = L x W. (Perimeter) P = 2L + 2W.

The children read and copy the notes together. Gauging that they are already familiar with the concept, AZ invites some students to go ahead with an activity sheet, measuring the area and perimeter of rectangles with increasing metric units. After reading the notes, AZ circles different words and terms, explains them in different ways and seeks other explanations from the children. For example, AZ explains that "calculate" is similar to "work out" and Sally suggests it's also like "figure out". The children also explore different ways of representing the formula, using different letters or words. For example, for area Adrian suggests, "space inside the rectangle = long line x short line" and writes it as "S = LL x SL".

When the children have annotated their notes on the basis of the discussion, AZ gives them time to learn the formulas and write them out from memory. The children then have time to explain how to find the area and perimeter of a rectangle to a friend and check their understanding. She uses this time to check for and modify understanding with individual students.

During the remaining week, AZ finds many incidental opportunities to integrate and check students' understanding of the content descriptor in-context. For example, she has students offer an impromptu estimation of the area and perimeter of a rectangle they have seen and then lets them check their estimates using a standard measure. When working on an ongoing community garden project, AZ gives the children time to think, pair, and share how area and perimeter calculations could be important for gardening.

Discussion

The illustrated differentiation of epistemic-pedagogic dispositions helps to clarify the pedagogical problems that arise from the dichotomisation or artificial balance of polarities in mathematics education, perhaps even in any domain, educative or otherwise. The weakness of binary oppositional dispositions is that they neglect their own weakness and the relative strengths and co-dependence of their relational opposites. For example, the Oppositional A scenario represents a deliberate pedagogical embrace of concrete, practical, holistic, student-centred ways of teaching; and subjective, interpretivist, and synthetic ways of knowing. HM engages the children through practical and authentic projects (e.g., the community garden) and does incidental mathematics (e.g., garden bed perimeter) along the way for children who display an intrinsic interest. The strength of the approach is its immediate concreteness. However, when

abstraction is sacrificed for, rather than explicitly derived from concreteness, it can paradoxically limit children's preparation to engage deeply with new and different concrete problems. In terms of the proficiency strands, the approach tends to immerse children in authentic but complex problems and reasoning without sufficiently scaffolding, or at least complementing, the immersion with conceptual knowledge, procedures, or de-cluttered representations. Our primary criticism of this oppositional approach is not that it may not work and be beneficial for some students; rather, it is that it unnecessarily excludes other positions from a pedagogical repertoire that may ultimately be used to engage more students over a longer period of time.

The counter-problem is of course illustrated in the Oppositional B scenario. Here, the pedagogical emphasis is on abstraction, with the presumption that knowledge of the abstraction will automatically be activated in multiple concrete contexts. So, DS uses rote methods, worksheets with abstract representations and theory-testing at the expense of more concretised pedagogies used in the Oppositional A scenario. The strength of the approach is in the immediate measurability and accumulation of content knowledge and its application to abstractions. However, when concreteness is sacrificed for abstraction, abstraction loses its meaning as there is little to abstract from or return to, in order to check the accuracy of the abstraction. On a broader scale, Ellis and Berry (2005) characterise similarly oppositional approaches to mathematics as progressive and traditional.

> Excellence, as defined by these models [traditional and progressive], meant either remembering rules and procedures with little concern for the connection of mathematics to students' lived experiences or, in the case of the progressives, focusing on the child's perceived interests or needs to the exclusion of being concerned with the learning of critical mathematical concepts. (p. 11)

While it is difficult and perhaps unnecessary to recognise these exclusions in a lesson, over time oppositional epistemic-pedagogic dispositions ironically contaminate the very ground they attempt to defend. Either, the abstraction becomes concretised in a particularly limited way (e.g., children become masters at calculating the area of concretely formless rectangles), or a dominant concretisation becomes an abstraction (e.g., children develop a disposition to associate rectangles with garden-beds). In terms of the proficiency strands, the approach tends to scaffold the students well in terms of conceptual knowledge and traditional procedure, but inhibit the adaptability, transferability and progressive application of these proficiencies for the resolution of "authentic" problems.

While perhaps a development in a cognitive sense, the equipositional dispositions can also be pedagogically destructive. They have the strength of drawing from both poles, but the weakness of ignoring the contextually dynamic and differentiated nature of "dynamic equilibriums". As such, the attempt to target learning through mechanically equal pedagogical divisions, is as effective as shooting arrows mechanically left, right, straight or randomly at, a randomly moving target. As such, RE's Year 5 classes' student time may turn into a "pooling of ignorance" for some students who do not adequately grasp the conceptual knowledge to apply it. Similarly, RE's "teacher time" may be wasted on other students who would better learn and appreciate the abstractions through the more concurrent exploration of concrete contexts. Likewise, SG's Year 5 class may be good at calculating the area and perimeter of rectangles in the school-yard with a single piece of grid paper with centimetre intervals, but struggle to move fluidly between different units of measurement or beyond the school-yard fence.

While each of these dispositions has its place, strengths and weaknesses, it is the parapositional disposition that encompasses most of these strengths and avoids most of the weakness, most of the time. Thus, AZ is relatively more responsive to the stability and dynamism of classroom contexts "gauging" and differentiating students' pedagogical needs, even within a single lesson, yet within a relatively stable scaffold. AZ provides concrete examples that flow between local and global contexts. She deliberately and explicitly derives abstractions from these

broad concretisations and encourages the children to do the same with new innovations and creations. She helps students to play and experiment with different representations to help them appreciate rather than replace, traditional representations that those in Ellis and Berry's (2005) "procedural-formalist paradigm" see as "an objective set of logically organized facts, skills, and procedures that have been optimised over centuries" (p 11). This is an approach to mathematics curriculum "that emphasises many physical models and representations—pictorial, manipulative, verbal, real-world, and symbolic—[and] is more successful in aiding students' development of conceptual understanding" (Reys et al, 2004, p. 285). AZ *draws on* and *draws out* students' prior knowledge to *draw them in* to formal learning. She applies in practice what Bobis et al (2013) describe in theory; "The realisation that children already possess a great deal of knowledge before formal instruction occurs has caused many educators to reconsider how children learn mathematics" (p. 6). In terms of the proficiency strands, this approach to teaching is both integrative and dissociative. Thus, *understanding*, *fluency*, *problem-solving* and *reasoning* are parts of the same whole that can be temporarily coordinated or separated for the most adaptable result. AZ's ways of being, knowing and teaching paradoxically sustain *and* dissolve binaries that offer navigational markers, albeit in the "wickedly complex" terrain of mathematics classrooms.

Conclusion

The introduction of the Australian Curriculum can reinvigorate the pedagogical dialogue in mathematics education, research and teacher professional development. We have argued that there is a relationship between a teacher's ways of knowing mathematics and his or her ways of teaching mathematics, and perhaps vice-versa. Our hope is to use some of the insights from research and theory in epistemological development and personal epistemology to contribute to a pedagogical dialogue that can often neglect the deeply personal *and* epistemological (i.e., personal epistemology) dimension of teaching and learning. Specifically, we hope that the binary-epistemic model can offer some conceptual clarity to the complex epistemic-pedagogic debates that sometimes escalate into "math wars".

We identified implicit tensions and oppositions in the literature on mathematics education that can inform how classroom teachers and teacher-educators interpret, discover and deliver content descriptors with and for their students. We identified binary oppositions such as concrete/abstract, subjective/objective, interpretivist/positivist, analytic/synthetic, transmission/discovery, behaviourist/constructivist, logic/intuition and theory/application and explored the affinities between the different constituents of pedagogical and epistemological binaries. Our aim is to encourage further research, dialogue and professional development to (*re*)conceptualise these oppositions and tensions as paradoxes that help to identify and (*re*)solve some of the wicked problems of classroom teaching. To further this dialogue we presented and illustrated a binary-epistemic model of development. We presented the parapositional approach as the most pedagogically and epistemologically dexterous, indeed ambidextrous - able *to teach* and *to know* from conceptual pole to conceptual pole, without being bound to one or the other, or to an artificially contextualised notion of balance or equality between the two.

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